Lasted edited: August 14, 2024

U of I MAC Handouts:

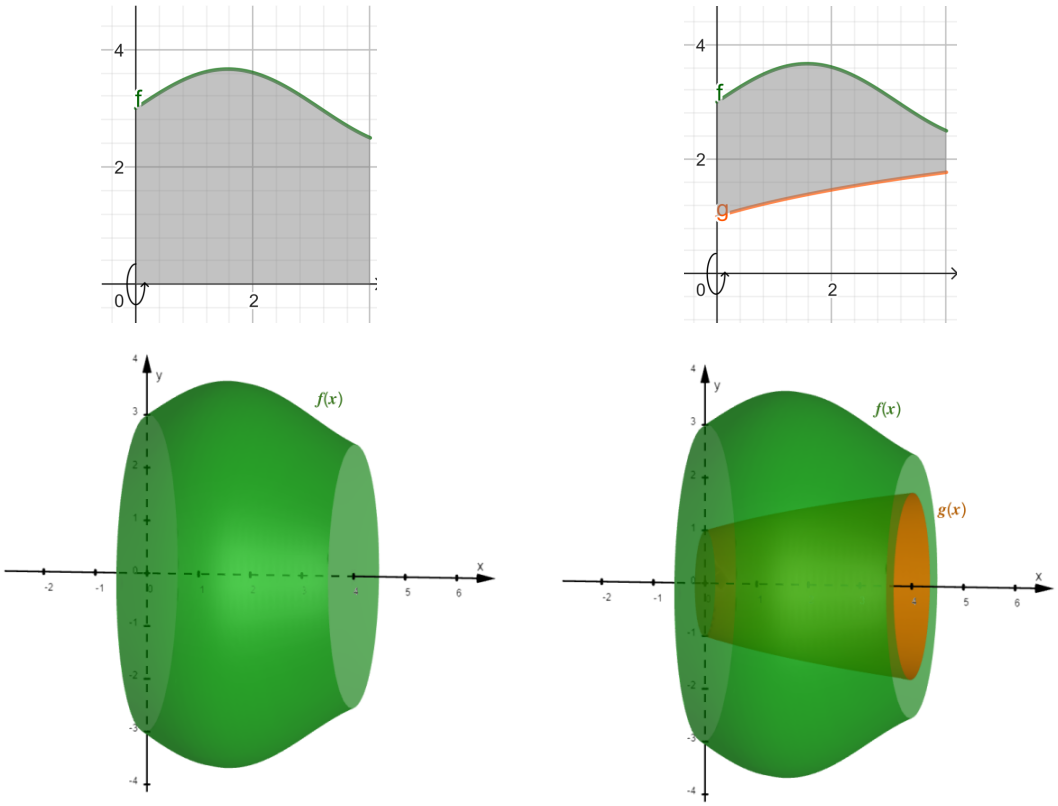
Solid of Revolutions Guide

Washer/Disk Method:

If a solid is symmetric around the x- or y- axis we can treat it as an infinite number of circles and find its volume by integrating a series of disks centered on the axis of symmetry

$$V = \int_a^b A(x)\,dx = \int_a^b \pi f^2(x)\,dx$$

because $f(x)$ is the radius of the circle ($A = \pi r^2$) at each point x along the shape. Notice the relation to the volume of a cylinder: $V = A \cdot h$, where A is the area of the base.



disk **washer**

This approach can be generalized for shapes with a gap in the center or "hollow" shapes. This is known as the washer method:

$$V = \pi \int_a^b ([f(x)]^2 - [g(x)]^2)\,dx$$

where $f(x)$ is the outer radius of the volume and $g(x)$ is the inner radius (see figure above). As before, we can swap y for x if the rotation is around the y-axis, calculating volume as:

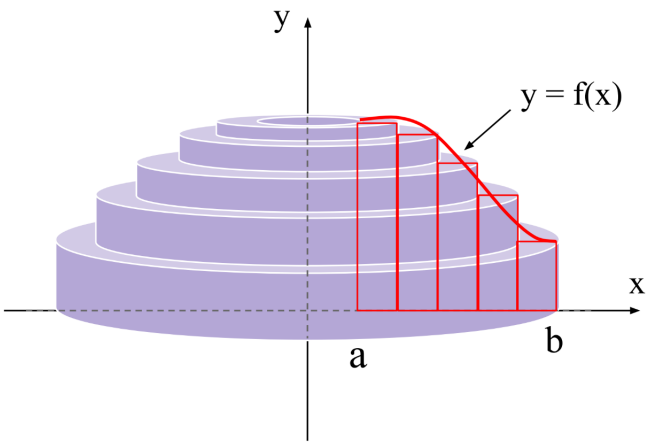
$$V = \pi \int_a^b ([f(y)]^2 - [g(y)]^2)\,dy$$

Shell Method:

If a solid is symmetric along the y-axis we can integrate a series of cylindrical shells centered on the y-axis from the inside of the solid to the outside along the x-axis (this also applies if axes are switched)

$$V = \int_a^b 2\pi x \, f(x) \, dx$$

$f(x)$ being the height of the cylinders, $2\pi x$ being the circumferences, dx being the thickness of the shells



Step-by-Step:

1. Identify the function(s) that define your shape.
2. Determine which method to use: if a function of x is rotated around the x-axis or a function of y around the y-axis, use the washer/disk method. If a function of x is rotated around the y-axis or a function of y around the x-axis, use the shell method.
3. Identify your a and b bounds.
4. Input the function(s) into a volume formula; make sure to use the volume formula that corresponds with the method you are using.
5. Solve the integral. If you have two functions and are using the washer method, be careful to square the functions first and *then* subtract.