



U of I MAC Handouts:

Rules for Logarithms, Exponents, and Radical

Logs to Exponentials

$$\log_b x = y \iff b^y = x$$

$$\ln x = y \iff e^y = x$$

Laws of Logarithms

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^r) = r \log_b x$$

Rules of Exponents

$$x^a \cdot x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

Rules of Radicals

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\sqrt[n]{b} = b^{\frac{1}{n}}$$

Change of Bases

$$\log_b(a) = \frac{\log(a)}{\log(b)}$$

Inverse Operations

$$e^{\ln x} = x$$

$$\ln(e^x) = x$$

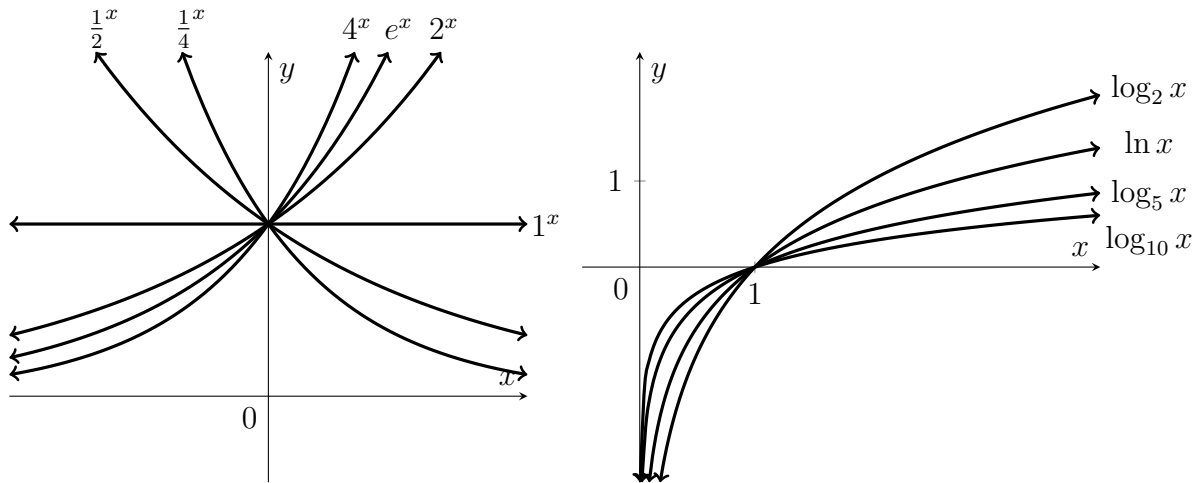
$$b^{\log_b x} = x$$

$$\log_b(b^x) = x$$

Domain and Range

- Logarithmic functions have a domain of $(0, \infty)$ and range of $(-\infty, \infty)$.
- Exponential functions have a domain of $(-\infty, \infty)$ and range of $(0, \infty)$.

Function Graphs



- The function $f(x) = b^x$ is *decreasing* when $b < 0$.
- The function $f(x) = b^x$ is *increasing* when $b > 0$.
- The function $f(x) = 1^x$ is a *horizontal line*.
- The larger the base of a logarithm, the *sharper* the curve.