

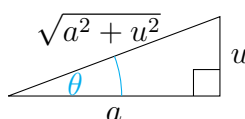


U of I MAC Handouts: Trigonometric Substitution Guide

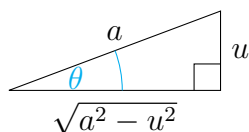
Common Substitutions to look for:		
Triangle 1	Triangle 2	Triangle 3
Look for: $\sqrt{a^2 + u^2}$ $a^2 + u^2$ Identity: $1 + \tan^2 \theta = \sec^2 \theta$	Look for: $\sqrt{a^2 - u^2}$ $a^2 - u^2$ Identity: $1 - \sin^2 \theta = \cos^2 \theta$	Look for: $\sqrt{u^2 - a^2}$ $u^2 - a^2$ Identity: $\sec^2 \theta - 1 = \tan^2 \theta$
Substitution: $u = a \tan \theta$ Expression for du : $du = a \sec^2 \theta d\theta$	Substitution: $u = a \sin \theta$ Expression for du : $du = a \cos \theta d\theta$	Substitution: $u = a \sec \theta$ Expression for du : $du = a \sec \theta \tan \theta d\theta$

*Note: It DOES matter where you have your constant, a , in the square roots, except in triangle 1

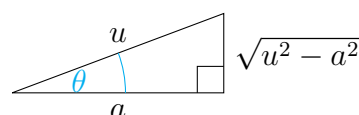
Common Right Triangles: (For the above table)



Triangle 1



Triangle 2

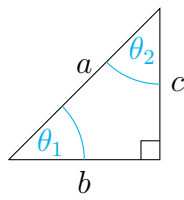


Triangle 3

Steps:

1. In order to determine which substitution is necessary, first look for terms with the square roots listed above
2. Identify which trigonometric function can be used; identify the value for x or u
3. Identify and take note of any constants that appear from u-substitutions
4. Find dx by taking the derivative of x , doing this gives an expression with $d\theta$
5. Perform substitution and rewrite integral in terms of theta and take note of any additional constants you may need to add to the integrand
6. Perform integration
7. Reverse the substitution and calculate area or add $+C$ as necessary

Trigonometry Review:



$$\begin{array}{ll} \sin \theta_1 = \frac{c}{a} & \sin \theta_2 = \frac{b}{a} \\ \cos \theta_1 = \frac{b}{a} & \cos \theta_2 = \frac{c}{a} \\ \tan \theta_1 = \frac{c}{b} & \tan \theta_2 = \frac{b}{c} \end{array}$$